Power & Surds (घात & करणी)

Law of exponent (घातांक का नियम) ⇒

(1) $x^a \times x^b = x^{a+b}$

(2) $x^a \div x^b = x^{a-b}$

(3) $(x^a)^b = x^{ab}$

(4) $\{\{x^a\}^b\}^c = x^{abc}$

(1) $x^a \times x^b \times x^c = 1$ then $a^3+b^3+c^3-3abc$ का मान

$x^{a+b+c} = x^0$ ⇓ 0

$a+b+c = 0$

(2) $\left(\frac{x^a}{x^b}\right)^c \times \left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b$

$\frac{x^{ac}}{x^{bc}} \times \frac{x^{ab}}{x^{ca}} \times \frac{x^{cb}}{x^{ab}} = 1$

(3) $2^x = 4^y = 8^z$ then find $x:y:z$

$2^x = 2^{2y} = 2^{3z}$

$\frac{x}{6} = \frac{2y}{6} = \frac{3z}{6}$

$x:y:z = 6:3:2$

(4) $2^x = 3^y = 6^z = k$ find relation b/w x, y & z

$2 = (k)^{1/x}$

$3 = (k)^{1/y}$

$6 = (k)^{1/z}$

$2 \times 3 = 6$

$(k)^{1/x} \times (k)^{1/y} = (k)^{1/z}$

$k^{\frac{1}{x}+\frac{1}{y}} = k^{\frac{1}{z}}$

$\boxed{\frac{1}{x}+\frac{1}{y}=\frac{1}{z}}$

(5) $x^a = y$, $y^b = z$, $z^c = x^3$ then $abc =$

$(x^a)^b = z$, $((x^a)^b)^c = x^3$

$x^{abc} = x^3 \Rightarrow \boxed{abc = 3}$

(6) $4^x \times 8^y = 2^{13}$, $3^{2x} \times 3^{2y} = 3^{10}$ then find x & y

$2^{2x} \times 2^{3y} = 2^{13}$

$2x + 3y = 13$

$2x + 2y = 10$

$\boxed{y = 3}$, $\boxed{x = 2}$

(7) $3^x - 3^y = 2^3 \times 3^{10}$ then find x

$3^{12} - 3^{10} = 3^{10}(3^2 - 1)$ $\Rightarrow x = 12, y = 10$

$3^{10} \times 2^3$

(8) $5^x - 5^y = 2^3 \times 3^1 \times 5^{10}$

$5^{12} - 5^{10} = 5^{10}(5^2 - 1) = 5^{10} \times 24$ $x = 12, y = 10$

$= 5^{10} \times 2^3 \times 3^1$

(9) $5^{\sqrt{x}} + 12^{\sqrt{x}} = 13^{\sqrt{x}}$

$5^2 + 12^2 = 13^2$

$\sqrt{x} = 2$

$\Rightarrow \boxed{x = 4}$

(10) $3^{\sqrt{x}} + 4^{\sqrt{x}} + 5^{\sqrt{x}} = 6^{\sqrt{x}}$

$3^3 + 4^3 + 5^3 = 6^3$

$\sqrt{x} = 3$

$\Rightarrow \boxed{x = 9}$

Q. $4^x \times 3^y = 144$ then find x & y.

$2^{2x} \times 3^y = 2^4 \times 3^2$

$2x = 4$, $y = 2$

$\Rightarrow x = 2$

Q. $3^x - 3^y = 2^3 \times 3^5$ then find x & y.

$= (3^2 - 1) \times 3^5$

$3^x - 3^y = 3^7 - 3^5$

$\Rightarrow x = 7, y = 5$

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(11) $\frac{6^8 \times 9^7 \times 10^5}{25^3 \times 3^{12} \times 4^6} = 2^x \times 3^y \times 5^z$ then $x+y+z$

$$\frac{2^8 \times 3^8 \times 3^{14} \times 2^5 \times 5^5}{5^6 \times 3^{12} \times 2^{12}} = \frac{2^{13} \times 3^{22} \times 5^5}{3^{12} \times 5^6 \times 2^{12}}$$

$$= 2^1 \times 3^{10} \times 5^{-1}$$

$x=1, y=10, z=-1$

$x+y+z = \underline{\underline{10}}$

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$(x)^{m/n} = [\sqrt[n]{x}]^m$ n (denometer) = Surds, m (Numinater) = Power

(12) $(32)^{4/5} = (\sqrt[5]{32})^4 = 2^4 = 16$

(13) $(16)^{-3/4} = \frac{1}{(16)^{3/4}} = \frac{1}{(\sqrt[4]{16})^3} = \frac{1}{2^3} = \frac{1}{8}$

(14) $\sqrt[3]{1024} = \sqrt[3]{8 \times 8 \times 8 \times 2} = 8\sqrt[3]{2}$

(15) $\sqrt[3]{500} = \sqrt[3]{5 \times 5 \times 5 \times 4} = 5\sqrt[3]{4}$

(16) $\sqrt[4]{768} = \sqrt[4]{4 \times 4 \times 4 \times 4 \times 3} = 4\sqrt[4]{3}$

$2^{10} = 1024$	$3^6 = 729$	$5^4 = 625$
$3^1 = 3$	$3^7 = 2187$	$5^5 = 3125$
$3^2 = 9$	$3^8 = 6561$	$5^6 = 15625$
$3^3 = 27$	$5^1 = 5$	
$3^4 = 81$	$5^2 = 25$	
$3^5 = 243$	$5^3 = 125$	

(17) $(9)^{1.5} = (9)^{3/2} = \left[\sqrt{9}\right]^3 = 3^3 = 27$

(18) $(256)^{.75} = (256)^{\frac{3}{4}} = \left(\sqrt[4]{256}\right)^3 = \left(\sqrt[4]{4\times4\times4\times4}\right)^3$

$= 4^3 = 64$

(19) $\sqrt[3]{27}$, $\sqrt[3]{27}$

$\Downarrow$ $\Downarrow$

$3\sqrt[3]{3\times3\times3}$ $\quad$ $\sqrt[3]{3\times3\times3}$

$= 9\sqrt{3}$ $\quad = 3$

(20) $\sqrt[3]{250} + \sqrt[3]{128} - \sqrt[3]{432} + \sqrt[3]{192} - \sqrt[3]{81} = a\sqrt[3]{2} + b\sqrt[3]{3}$

then find $a+b$

Sol → $5\sqrt[3]{2} + 4\sqrt[3]{2} - 6\sqrt[3]{2} + 4\sqrt[3]{3} - 3\sqrt[3]{3}$

$\Rightarrow \quad 3\sqrt[3]{2} + \sqrt[3]{3}$

$a = 3, \quad b = 1 \Rightarrow \quad a+b = 4$

(21) $\dfrac{\sqrt{48} + \sqrt{32}}{\sqrt{75} + \sqrt{50}} = \dfrac{4\sqrt{3} + 4\sqrt{2}}{5\sqrt{3} + 5\sqrt{2}}$

$= \dfrac{4(\sqrt{3} + \sqrt{2})}{5(\sqrt{3} + \sqrt{2})}$

$= \dfrac{4}{5}$

(22) $\dfrac{7}{3+\sqrt{2}}$ का दशमलव के तीन अंको तक मान

$\dfrac{7}{3+\sqrt{2}} \times \dfrac{3-\sqrt{2}}{3-\sqrt{2}} = \dfrac{7(3-\sqrt{2})}{7} = 3 - 1.414 = 1.586$

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(23) $\sqrt{\frac{\sqrt{3}-1}{\sqrt{3}+1}} + \sqrt{3}+1$ का मान

$\sqrt{\frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}} + \sqrt{3}+1 = \sqrt{\frac{3-\sqrt{3}-\sqrt{3}+1}{3-1}} + \sqrt{3}+1$

$= \sqrt{\frac{4-2\sqrt{3}}{2}} + \sqrt{3}+1 = \sqrt{2-\sqrt{3}+\sqrt{3}+1} = \sqrt{3}$

(24) $\sqrt{\frac{\sqrt{3}+1}{\sqrt{3}+\sqrt{2}}} + \frac{6}{\sqrt{6}} - 1$ का मान

$\sqrt{\frac{\sqrt{3}+1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}} + \frac{6}{\sqrt{6}} - 1$

$\sqrt{3+\sqrt{3}-\sqrt{6}-\sqrt{2}+\sqrt{6}-1} = \sqrt{2+\sqrt{3}-\sqrt{2}}$

(25) $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+2} + \frac{1}{2+\sqrt{5}} + \cdots\cdots + \frac{1}{\sqrt{99}+10}$ का मान

$\sqrt{2}-1+\sqrt{3}-\sqrt{2}+2-\sqrt{3}+\sqrt{5}-2+\cdots\cdots+10-\sqrt{99}$

$10-1 = 9$

(26) $\frac{2+\sqrt{3}}{\sqrt{3}+\sqrt{2}} + \frac{2-\sqrt{3}}{\sqrt{3}-\sqrt{2}}$ का मान

$\frac{(2+\sqrt{3})(\sqrt{3}-\sqrt{2}) + (2-\sqrt{3})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$

$= \frac{2\sqrt{3}-2\sqrt{2}+3-\sqrt{6}+2\sqrt{3}+2\sqrt{2}-3-\sqrt{6}}{3-2} = 2\sqrt{3}-2\sqrt{2}+3-\sqrt{6}+2\sqrt{3}+2\sqrt{2}-3-\sqrt{6}$

$= 4\sqrt{3}-2\sqrt{6}$

$= 2(2\sqrt{3}-\sqrt{6})$

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1) $\frac{\sqrt{5}+1}{2-\sqrt{3}} = a+b\sqrt{3}+c\sqrt{5}+d\sqrt{15}$ then find $a+b+c+d$

$\frac{\sqrt{5}+1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = 2\sqrt{5}+\sqrt{15}+2+\sqrt{3} = a+b\sqrt{3}+c\sqrt{5}+d\sqrt{15}$

$\Rightarrow a=2, b=1, c=2, d=1$

$a+b+c+d = \underline{\underline{6}}$

8) $\frac{1}{\sqrt[3]{4}+\sqrt[3]{2}+1} = a\sqrt[3]{4}+b\sqrt[3]{2}+c$ then find $a+b+c$

$\frac{1}{(\sqrt[3]{2})^2+(\sqrt[3]{2}+1)} \times \frac{(\sqrt[3]{2}-1)}{(\sqrt[3]{2}-1)} = \frac{\sqrt[3]{2}-1}{1}$

$\Rightarrow a=0, b=1, c=-1 \quad a+b+c = \underline{\underline{0}}$

$\frac{1}{(x^2+x+1)} \times \frac{(x-1)}{(x-1)} = \frac{x-1}{x^3-1}$

$a^4+a^2b^2+b^4 = (a^2+ab+b^2)(a^2-ab+b^2)$

9) $(\sqrt{a}+\sqrt{b}+\sqrt{c}-\sqrt{d})(\sqrt{a}-\sqrt{b}+\sqrt{c}-\sqrt{d})$ का मान एक परिमेय संख्या है जहाँ $\sqrt{a}, \sqrt{b}, \sqrt{c}, \sqrt{d}$ etc. सभी अपरिमेय संख्या है तब a, b, c तथा d का संबंध ज्ञात करो। जहाँ $\sqrt{ab}, \sqrt{bc}, \sqrt{ca}, \sqrt{da}$ etc. सभी imperfect square है।

$(a+d)-(b+c)$

(30) $x = \frac{\sqrt{3}}{2}$ then $\sqrt{1+x} + \sqrt{1-x}$, $\sqrt{1+x} - \sqrt{1-x}$ का मान

$$\sqrt{1+\frac{\sqrt{3}}{2}} = \sqrt{\frac{2+\sqrt{3}}{2} \times \frac{2}{2}} = \sqrt{\frac{4+2\sqrt{3}}{4}} = \sqrt{\frac{(\sqrt{3})^2 + 2(\sqrt{3})(1) + 1^2}{4}}$$

$$= \sqrt{\frac{(\sqrt{3}+1)^2}{4}} = \frac{\sqrt{3}+1}{2}$$

(i) $\frac{\sqrt{3}+1}{2} + \frac{\sqrt{3}-1}{2} = \frac{\sqrt{3}+1+\sqrt{3}-1}{2} = \sqrt{3}$

(ii) $\frac{\sqrt{3}+1}{2} - \frac{\sqrt{3}-1}{2} = \frac{\sqrt{3}+1-\sqrt{3}+1}{2} = 1$

or $\sqrt{1+x} + \sqrt{1-x} = k$

$1+x+1-x+2\sqrt{1-x^2} = k^2$

$2 + 2\sqrt{1-\frac{3}{4}} = k^2 \Rightarrow 2 + 2 \times \frac{1}{2} = k^2 \Rightarrow k^2 = 3 \Rightarrow k = \sqrt{3}$

(31) $x = 4 + 2\sqrt{3}$ then $\sqrt{x} + \frac{1}{\sqrt{x}}$ का मान

$\sqrt{x} + \frac{1}{\sqrt{x}} = k \Rightarrow x + \frac{1}{x} + 2 = k^2 \Rightarrow 4 + 2\sqrt{3} + \frac{1}{4+2\sqrt{3}} + 2 = k^2$

$\Rightarrow \frac{4+2\sqrt{3}}{1} + \frac{4-2\sqrt{3}}{4} + \frac{2}{1} = k^2$

$\Rightarrow \frac{16+8\sqrt{3}+4-2\sqrt{3}+8}{4} = k^2$

$\Rightarrow \frac{28+6\sqrt{3}}{4} = k^2$

$\Rightarrow k = \sqrt{\frac{28+6\sqrt{3}}{4}} = \sqrt{\frac{(3\sqrt{3}+1)^2}{4}} = \frac{3\sqrt{3}+1}{2}$

Alt. $x = (\sqrt{3}+1)^2$

$\sqrt{x} = (\sqrt{3}+1)$

$\frac{1}{\sqrt{x}} = \frac{\sqrt{3}-1}{2}$

$\sqrt{3}+1 + \frac{\sqrt{3}-1}{2} \Rightarrow \frac{3\sqrt{3}+1}{2}$

(32) $x = 7 + 2\sqrt{10}$ then $\sqrt{x} + \frac{1}{\sqrt{x}}$ का मान

$\sqrt{x} = \sqrt{7 + 2\sqrt{10}} = \sqrt{(\sqrt{5}+\sqrt{2})^2} = \sqrt{5}+\sqrt{2}$

$2\sqrt{5}\sqrt{2}$

$\sqrt{5}+\sqrt{2} + \frac{1}{\sqrt{5}+\sqrt{2}} = \sqrt{5}+\sqrt{2} + \frac{\sqrt{5}-\sqrt{2}}{3} = \frac{3\sqrt{5}+3\sqrt{2}+\sqrt{5}-\sqrt{2}}{3}$

$= \frac{4\sqrt{5}+2\sqrt{2}}{3} = \frac{2}{3}(2\sqrt{5}+\sqrt{2})$

(33) $\sqrt{\sqrt{3} + \sqrt{7 - \sqrt{48}}}$

$4\sqrt{3}$

$2\times 2\times\sqrt{3}$

$\sqrt{\sqrt{3} + \sqrt{(2-\sqrt{3})^2}} = \sqrt{\sqrt{3}+2-\sqrt{3}} = \sqrt{2}$

(34) $\frac{2}{\sqrt{12-\sqrt{140}}} - \frac{1}{\sqrt{11-\sqrt{120}}}$ का मान

$\frac{2}{\sqrt{12-2\sqrt{35}}} - \frac{1}{\sqrt{11-2\sqrt{30}}}$

$2\sqrt{7}\sqrt{5}$ $\quad 2\sqrt{6}\sqrt{5}$

$\frac{2}{\sqrt{(\sqrt{7}-\sqrt{5})^2}} - \frac{1}{\sqrt{(\sqrt{6}-\sqrt{5})^2}} = \frac{2}{\sqrt{7}-\sqrt{5}} - \frac{1}{\sqrt{6}-\sqrt{5}}$

$= \sqrt{7}+\sqrt{5} - (\sqrt{6}+\sqrt{5})$

$= \sqrt{7}-\sqrt{6}$

Nested roots ⇒

$\left\{\sqrt[n]{k\sqrt[n]{k\sqrt[n]{k\sqrt[n]{k}}}\ldots\ldots\infty}\right\}^n = \{x\}^n \Rightarrow kx = x^n$

$x^{n-1} = k \Rightarrow x = \sqrt[n-1]{k}$

$\sqrt{k\sqrt{k\sqrt{k\sqrt{k}}}\ldots n} = (k)^{\frac{2^n-1}{2^n}}$

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(35) $\sqrt[2]{3\sqrt[2]{3\sqrt[2]{3\sqrt{3}}}}\ldots\infty$

$= 3^{\frac{1}{2-1}} = 3 \quad \left[\sqrt[n]{3} = 3\right]$

(36) $\sqrt[3]{4\sqrt[3]{4\sqrt[3]{4\sqrt[3]{4}}}}\ldots\infty$

$x = 4^{\frac{1}{3-1}} = 4^{1/2} = \sqrt{4} = 2 \quad \left\{\sqrt[3-1]{4} \quad \sqrt[2]{4} = \sqrt{4} = 2\right\}$

(37) $\sqrt{4\sqrt{4\sqrt{4\sqrt{4\sqrt{4}}}}} = 2^n$ then n का मान

$(4)^{\frac{2^5-1}{2^5}} = 4^{\frac{31}{32}} = \left(2^2\right)^{\frac{31}{32}} = 2^{\frac{31}{16}} = 2^n$

$\Rightarrow n = \frac{31}{16}$

(38) $\sqrt[3]{4\sqrt{2\sqrt[3]{4\sqrt{2\sqrt[3]{4\sqrt{2}}}}}}\ldots\infty$ का मान

$\left(\sqrt[3]{4\sqrt{2x}}\right)^3 = x^3$

$\left\{4\sqrt{2x}\right\}^2 = \left\{x^3\right\}^2$

$16 \times 2x = x^6$

$\Rightarrow 32 = x^5 \Rightarrow \boxed{x = 2}$

(39) $\sqrt[3]{4\sqrt[3]{4\sqrt[3]{4\sqrt[3]{4}}}}$

$4^{\frac{1}{3}} \cdot 4^1 = 4^{\frac{4}{3}} \quad \left(4^{4/3}\right)^{1/3} = 4^{4/9} \cdot 4^1 = \left(4^{\frac{13}{9}}\right)^{1/3} = 4^{\frac{13}{27}} \cdot 4^1$

$= \left(4^{\frac{40}{27}}\right)^{1/3} = 4^{\frac{40}{81}}$

$\Rightarrow \sqrt[3]{k\sqrt[3]{k\sqrt[3]{k\sqrt[3]{k}}}}$

$k \cdot k^{1/3} = \left(k^{4/3}\right)^{1/3} = k^{\frac{4}{9}} \cdot k^1 = \left(k^{\frac{13}{9}}\right)^{1/3}$

$= k^{\frac{13}{27}} \cdot k^1 = \left(k^{\frac{40}{27}}\right)^{1/3}$

$= k^{\frac{40}{81}}$

$\sqrt[n]{k\sqrt[n]{k\sqrt[n]{k\sqrt[n]{k}}}}\ldots\infty = (K)^{\frac{5^n-1}{2\times 5^n}}$

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(40) $\sqrt[3]{4\sqrt[3]{4\sqrt[3]{4\sqrt[3]{4}}}} = 2^n$

$4 \cdot 4^{\frac{1}{3}} = \left(4^{\frac{4}{3}}\right)^{\frac{1}{4}} = 4^{\frac{1}{3}} \cdot 4 = \left(4^{\frac{4}{3}}\right)^{\frac{1}{2}} = 4^{\frac{2}{3}}$

$4^{\frac{2}{3}} \cdot 4 = \left(4^{\frac{5}{3}}\right)^{\frac{1}{3}} = 4^{\frac{5}{9}}$

$= 2^{\frac{10}{9}}$

$\boxed{n = \frac{10}{9}}$

(41) $\sqrt{k \pm \sqrt{k \pm \sqrt{k \pm \sqrt{k \pm \ldots\ldots\infty}}}} = \frac{\sqrt{4k+1} \pm 1}{2}$

$\sqrt{k+x} = x$

$k + x = x^2$

$x^2 - x - k = 0$

$x = \frac{-(-1) \pm \sqrt{1+4k}}{2} = \frac{\sqrt{4k+1} + 1}{2}$

(42) $\sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots\ldots\infty}}}}$

$\frac{\sqrt{4\times 12+1} + 1}{2} = \frac{\sqrt{49}+1}{2} = \frac{7+1}{2} = \frac{8}{2} = 4$

(43) $\sqrt{12 - \sqrt{12 - \sqrt{12 - \sqrt{12 - \ldots\ldots\infty}}}}$

$\frac{\sqrt{4\times 12+1} - 1}{2} = \frac{7-1}{2} = \frac{6}{2} = 3$

$12 = 3 \times 4$

छोटा वाला (-) Series का Answer

बड़ा वाला (+) Series का Answer

(44) $\sqrt{2 - \sqrt{2 - \sqrt{2 - \sqrt{2 - \ldots\ldots\infty}}}}$ 2×1

$\frac{\sqrt{4\times 2+1} - 1}{2} = \frac{\sqrt{9}-1}{2} = \frac{3-1}{2} = \frac{2}{2} = 1$

$$\begin{cases} \sqrt{K+\sqrt{K+\sqrt{K+\sqrt{K+\ldots\infty}}}} - \sqrt{K-\sqrt{K-\sqrt{K-\sqrt{K-\ldots\infty}}}} = 1 \\ \frac{\sqrt{4K+1}+1}{2} - \frac{\sqrt{4K+1}-1}{2} = \frac{\sqrt{4K+1}+1-\sqrt{4K+1}+1}{2} = \frac{2}{2} = \end{cases}$$

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(45) $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}\ldots\ldots\infty$

2×3

$$\frac{\sqrt{4\times6+1}+1}{2} = \frac{5+1}{2} = \frac{6}{2} = 3$$

(46) $\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1}}}}\ldots\ldots\infty$

$$\frac{\sqrt{4\times1+1}+1}{2} = \frac{\sqrt{5}+1}{2}$$

(47) $x = 5+\sqrt{21}$ then $\sqrt{\frac{x}{2}} + \sqrt{\frac{2}{x}}$ का मान

$$\frac{x}{2} + \frac{2}{x} + 2 = K^2$$

$$\frac{5+\sqrt{21}}{2} + \frac{2}{5+\sqrt{21}} + 2 = K^2$$

$$\frac{(5+\sqrt{21})^2 + 4 + 4(5+\sqrt{21})}{2(5+\sqrt{21})} = K^2$$

$$\frac{25+21+10\sqrt{21}+4+20+4\sqrt{21}}{10+2\sqrt{21}} = K^2$$

$$\frac{70+14\sqrt{21}}{10+2\sqrt{21}} = K^2$$

$$\frac{7(10+2\sqrt{21})}{(10+2\sqrt{21})} = K^2$$

$\Rightarrow K^2 = 7$

$\Rightarrow K = \sqrt{7}$

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(48) $(a-1)\sqrt{3} + b\sqrt{2} = \sqrt{3} - \sqrt{2}$ then $a+b$ का मान

$a-1 = 1 \Rightarrow a = 2$

$b = -1$

$\Rightarrow a+b = 2-1 = 1$

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(49) $a = \frac{1}{2+\sqrt{3}}$, $b = \frac{1}{3-2\sqrt{2}}$ then $\frac{1}{a+1} + \frac{1}{b+1}$ का मान

$a = 2-\sqrt{3} \Rightarrow a+1 = 3-\sqrt{3}$

$b = 3+2\sqrt{2} \Rightarrow b+1 = 4+2\sqrt{2}$

$$\frac{1}{3-\sqrt{3}} + \frac{1}{4+2\sqrt{2}} = \frac{3+\sqrt{3}}{6} + \frac{4-2\sqrt{2}}{8} = \frac{12+4\sqrt{3}+12-6\sqrt{2}}{24}$$

$$= \frac{24+4\sqrt{3}-6\sqrt{2}}{24} = 2\left(\frac{12+2\sqrt{3}-3\sqrt{2}}{24}\right)$$

$$= \frac{12+2\sqrt{3}-3\sqrt{2}}{12}$$

(50) $\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}} + \frac{1}{\sqrt{2}-\sqrt{3}+\sqrt{5}}$ का मान

$$\frac{\sqrt{2}-\sqrt{3}+\sqrt{5}+\sqrt{2}+\sqrt{3}-\sqrt{5}}{(\sqrt{2}+\sqrt{3}-\sqrt{5})(\sqrt{2}-\sqrt{3}+\sqrt{5})}$$

$$= \frac{2\sqrt{2}}{2-\sqrt{6}+\sqrt{10}+\sqrt{6}-3+\sqrt{15}-\sqrt{10}+\sqrt{15}-5}$$

$$= \frac{2\sqrt{2}}{2\sqrt{15}-6} = \frac{\sqrt{2}}{\sqrt{15}-3} \times \frac{\sqrt{15}+3}{\sqrt{15}+3}$$

$$= \frac{\sqrt{2}(\sqrt{15}+3)}{15-9}$$

$$= \frac{\sqrt{2}(\sqrt{15}+3)}{6\ 3\sqrt{2}}$$

$$= \frac{\sqrt{3}(\sqrt{5}+\sqrt{3})}{\sqrt{3}\,3\sqrt{2}} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{6}}$$

Power का H.C.F. Surds के रूप में use करते हैं।

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Comparision of Surds :-

(51) $\sqrt[3]{2}, \sqrt[4]{4}, \sqrt[6]{5}$

$(\sqrt[3]{2})^{12}, (\sqrt[4]{4})^{12}, (\sqrt[6]{5})^{12}$

$2^4, 4^3, 5^2$

$16, 64, 25$

(i) (ii) (iii) $\quad$ (ii) > (iii) > (i)

(52) $\sqrt{11}-\sqrt{7}, \sqrt{15}-\sqrt{11}, \sqrt{13}-\sqrt{9}$

$\frac{4}{\sqrt{11}+\sqrt{7}}, \frac{4}{\sqrt{15}+\sqrt{11}}, \frac{4}{\sqrt{13}+\sqrt{9}}$

(i) (ii) (iii) $\quad$ (iii) > (ii) > (i)

(53) $\sqrt{11}+2, \sqrt{7}+\sqrt{8}, \sqrt{13}+\sqrt{2}$

$\sqrt{11}+\sqrt{4}, \sqrt{7}+\sqrt{8}, \sqrt{13}+\sqrt{2}$

$15+2\sqrt{44}, 15+2\sqrt{56}, 15+2\sqrt{26}$

(ii) (i) (iii)

(54) $\sqrt{169}+\sqrt{1.69}+\sqrt{0.0169}+\sqrt{0.000169}$

$13+1.3+0.13+0.013 = 14.443$

वर्गमूल ज्ञात करना

2 | 500	22.35
2 | 4	
42 | 100	
2 | 84	
443 | 1600	
3 | 1329	
4465 | 27100	

Short

22^2 $\quad$ 23^2

484 \| 529

45

22.35

(1) $2^{30}, 3^{25}, 5^{15}$ को बढ़ते क्रम में लिखें

$(2^{30})^{1/5}, (3^{25})^{1/5}, (5^{15})^{1/5}$

$2^6, 3^5, 5^3$

$64, 243, 125$

$2^{30} < 5^{15} < 3^{25}$

(2) $2^{30}, 3^{15}, 5^8$

$2^{30}, 3^{15}$

$(2^{30})^{1/15}, (3^{15})^{1/15}$

$2^2 > 3$

$3^{15} \quad 5^8$

$3^8 \quad 5^8$

$2^{30} \quad 5^8$

$(2^{30})^{1/2} \quad (5^8)^{1/2}$

$2^{15} > 5^4$

$2^{30} > 3^{15} > 5^8$

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(55) $2^x = x^2$ then find all possible value of x

$x = 2, 4$

(56) $20^{\sqrt{x}} + 21^{\sqrt{x}} = 29^{\sqrt{x}}$

$\sqrt{x} = 2 \Rightarrow x = 4$

(57) a, b तथा c तीन क्रमागत पूर्णांक हैं जहाँ $a < b < c$ है तथा $a^2 + b^2 = c^2$ तब $a+b+c$ ज्ञात करें।

हल → $a = x-1,\ b = x,\ c = x+1$

$(x-1)^2 + x^2 = (x+1)^2$

$x^2 - 2x + 1 + x^2 = x^2 + 2x + 1$

$x^2 = 4x$

$x(x-4) = 0 \Rightarrow x = 0, 4$

$\underset{a+b+c}{\text{Sum}} = 0$

$x = 0$ पर $a = -1,\ b = 0,\ c = 1$

$x = 4$ पर $a = 3,\ b = 4,\ c = 5$

$\underset{a+b+c}{\text{Sum}} = 12$

1	370	19.23
1	1	
29	270	
9	261	
382	900	
2	764	
3843	1360	
3	11529	

19^2 370 20^2

361 400

39

19.23

(58) $x = 5 + \sqrt{21}$ then $\sqrt{x}$ का मान

$2x = 10 + 2\sqrt{21}$

$2 \times \sqrt{7} \times \sqrt{3}$

$\sqrt{2x} = \sqrt{(\sqrt{7}+\sqrt{3})^2}$

$\sqrt{2x} = \sqrt{7} + \sqrt{3}$

$\sqrt{x} = \frac{\sqrt{7}+\sqrt{3}}{\sqrt{2}}$

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Ex. $(3x+2y)^4$ में कुल कितने पद है, (b) xy के गुणांक (c) सभी पदों के गुणांकों का योग

$\Downarrow$

5

$\{(a+b)^n$ में no of term $= n+1\}$

(a) $(9x^2+12xy+4y^2)$ में xy का गुणांक $= 0$

$81x^4+144x^2y^2+16y^4+216x^3y+96xy^3+72x^2y^2$

$\Rightarrow 81x^4+216x^2y^2+16y^4+216x^3y+96xy^3$

$81+216+16+216+96 = 625$

सभी पदो के गुणांको का मान $x=y=1$ $\quad (3+2)^4 = 5^4 = 625$

Ex. $(2x-3y-4)(2x-3y+4) = ax^2+by^2+cxy+dx+ey+f$

$4\times 12 = 48$

$2\times1-3\times(-2)-4 \quad 2\times1-3\times(-2)+4$

$2\times1-3\times1-4 \quad 2\times1-3\times1+4$

$1\times-2 \quad 1\times-2$

$a+b+c+d+f$

$-5\times3 = 15$

$a+4b-2c+d-2e+f$

1×3

$x=1, y=-2$

Ex. $\sqrt{13+\sqrt{139+\sqrt{17+\sqrt{64}}}} = 5$

8

$\sqrt{25} = 5 \qquad \sqrt{139+5} = \sqrt{144} = 12 \qquad 13+12 = 25$

$\sqrt{25} = 5$

Ex. $\sqrt{\frac{3}{2}}$ का दशमलव के बाद दूसरा अंक

$\sqrt{\frac{3}{2}\times\frac{2}{2}} = \frac{\sqrt{6}}{2}$

$= \frac{2.44}{2}$

$= 1.22$

2	6	2.44
2	4	
44	200	
4	176	
484	2400	
4		